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Developing Algebraic Thinking in Early Grades: Case Study of Korean Elementary School Mathematics[1]

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Abstract*:* This study presents a case study of the Korean elementary mathematics curriculum on algebraic thinking. After describing a general overview of mathematics education in Korea, this study identifies six kinds of mathematical abilities related to algebra emphasized in Korean elementary textbooks: generalization, abstraction, analytic thinking, dynamic thinking, modeling, and organization. Based on the position that as a way of thinking algebra is much more than the set of techniques and knowledge, Korean elementary textbooks provide various activities under the goals to improve above six kinds of thinking. This study introduces sample problems for each specific goal and analyzes content coverage for algebraic thinking each grade level. Finally, this study suggests new directions for promoting algebraic thinking.

Introduction

Korean students begin the formal algebra course in the 7th grade. As a subject dealing with symbols, algebra is very important in that most concepts in secondary school mathematics cannot be easily learned without understanding of symbols. The importance of algebra becomes clear in considering the history of mathematics to show that without the use of symbols by Viete, analytic geometry by Descartes and calculus by Newton and Leibniz would have been almost impossible to develop.

Algebra seems to be far more difficult for students than expected. Research shows that the concepts of variables and functions make serious epistemological obstacles for many students and even teachers (Tall, 1991; Toshiakaira, 2003). These obstacles are understandable in that symbol use in mathematics results from a 3000 year history since the Babylonian era rather than the sudden occurrence due to Viete's genius.

Korea provides various instructional efforts for students to learn algebra more easily. In junior high school, when algebra is introduced, many concrete operational activities are given to reduce a cognitive gap between algebra and arithmetic. Also, in elementary school, many prerequisite activities are provided to establish a good foundation for learning algebra. This paper introduces Korean perspectives for

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improving algebraic thinking in grades 1-6 under the 7th national curriculum which has been implemented since 2000.

Overview of Korean Mathematics Education

The nature of the 7th national curriculum

The ultimate goal of the 7th mathematics curriculum is to cultivate students with a creative and autonomous mind by achieving the following three main aims: first, to understand basic mathematical concepts and principles through the concrete experiences with various manipulative materials and the use of daily life phenomena related with mathematics; second, to foster mathematical modeling abilities through solving various problems posed within and without mathematics; and third, to keep a positive attitude about mathematics and mathematics learning through emphasizing a connection between mathematics and the real world (Lew, 1998).

The new curriculum, emphasizing basic mathematical knowledge and the *practical* application of mathematics, contrasts sharply with the 3rd curriculum, which emphasized "mathematical structure" in a context of *theoretical* mathematics. The 4th curriculum issued in 1980 revised the 3rd curriculum which had strongly accepted the philosophy of "New Math". For about 20 years now, since 1980, resulting from the influence of NCTM, a basic position of mathematics education has slowly changed from a *theoretical* to a *practical* focus and includes components of problem solving, application and the use of technology.

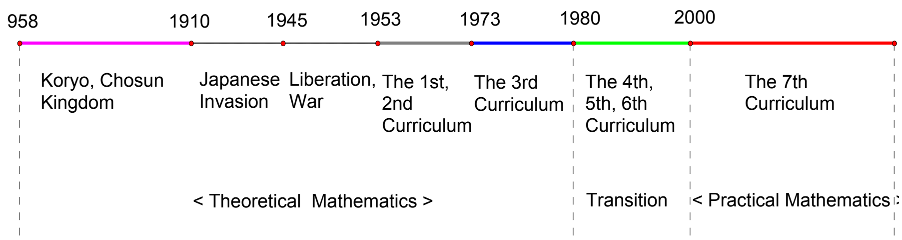

Figure 1. History of mathematics curriculum of Korea

In the 7th mathematics curriculum, students are expected to be able to organize real world phenomena mathematically, to determine mathematical relations of concepts and principles by the process of abstraction based on their own concrete operations, to promote mathematical reasoning abilities through solving various problems by using mathematical knowledge and skills they have already acquired, and finally to acquire the positive attitude toward mathematics.

Traditionally, the most serious problem in Korean mathematics education is that mathematics is considered just a tool subject for students to prepare for college entrance examination in which *theoretical* mathematics focusing on mathematical knowledge is emphasized more than *practical* mathematics focusing on its utility. The emphasis on the examination without internal motivation for learning has made it difficult for students to have real understanding and to develop reasonable and productive thinking abilities. It demands students accept mechanically undigested content organized around topics frequently appearing in the examination. As a result, it becomes almost impossible for students to nurture an investigative attitude and a desirable mental habit.

Korea has a long history of emphasizing theoretical mathematics for a selective examination (See Figure 1). In 958AD the "Koryeo" Kingdom (918-1392AD) introduced mathematics into its examination system to select government officials (Needham, 1954, p.139). Whoever wanted to become a government official ought to have learned many Chinese mathematical classics including "Chui-chang Suan Shu"(nine Chapters on the mathematical Art) for 7-9 years in the national schools. This tradition of mathematics education continued during the period of the Koryeo Kingdom and "Chosun" Kingdom (1392 - 1910AD).

Instructional and evaluation methods emphasized in the 7th curriculum

The 7th curriculum emphasizes various types of instruction to improve efficiency and significance of students' mathematical learning. It recommends that students should be able to experience the joy of discovery and maintain their interest in mathematics by pursuing the following instructional methods in their classrooms:

- to emphasize concrete operational activities in order to help students to discover principles and rules and solve problems embedded in such a discovery;
- to have students practice basic skills to help students be familiar with them and problem-solving abilities in order to use mathematics in their everyday life;
- to present concepts and principles in the direction from the concrete to the abstract in order to activate self-discovery and creative thinking;
- to induce students to recognize and formulate problems from situations both within and outside mathematics;
- to select appropriate questions and subsequently provide feedback in a constructive way in order to consider the stages of students' cognitive development and experiences;
- to use open-ended questions in order to stimulate students' creativity and divergent thinking;
- to value the application of mathematics in order to foster a positive attitude toward mathematics;

- to help students understand the problem-solving process; and
- to use basic problem-solving strategies in order to enhance students' problem-solving abilities.

The 7th curriculum emphasizes that mathematical power should be evaluated by realizing the following methods in their classrooms:

- to emphasize processes more than products in order to foster students' thinking abilities;
- to focus on students' understanding of a problem and the problem-solving process as well as its results in order to evaluate students' problem- solving abilities;
- to focus on students' interests, curiosity and attitudes toward mathematics in order to evaluate students' mathematical aptitudes;
- to focus on student's abilities to think and solve problems in a flexible, diverse and creative fashion in order to evaluate mathematical learning; and
- to use a variety of evaluation techniques such as extended- response questions, observations, interviews as well as multiple-choices in order to evaluate students' mathematical learning.

Instructional Efforts for Remodeling Mathematics Classes

In the late 1990s, there were three kinds of instructional movements to remodel the mathematics classes: Open education, use of technology, and performance assessment. Although these movements encountered resistance due to various barriers of human resources and environmental factors, they have contributed critically to the development of instructional methods. As well, they had an effect on the development of the 7th national curriculum.

"Open" education

Teacher-centered, whole-class lessons have been criticized as hindering students' thinking abilities and attitudes toward mathematics learning. It is identified as the main culprit of the "total crisis" in mathematics education. For several years in the late of 1990s, the Korean mathematics education community was filled with debate of "open education movement" to improve students' thinking and communication abilities and to provide them with the joy of doing mathematics.

There was a spontaneous movement of teachers in favour of remodeling mathematics classes against the "closed" school system that erased students' creativity and autonomy by preparing them only for college entrance examination. Teachers provided various materials including investigation tasks, discussion topics, strategy games and projects, and adopted small-group, cooperative learning to cultivate cooperative spirit and facilitate discussion between peers.

This movement has faced many barriers such as teachers' instructional competency, cultural conflicts with a large number of "traditional" teachers, an absence of the appropriate teaching model, a shortage of good materials, class size of over 40, and a conventional school system. However, it has contributed to change dramatically the instructional methods of most teachers.

Technology

Recently, the Korean mathematics education community has been encouraged by the instructional potentiality of various computer software, such as LOGO, designed to explore mathematics in a micro-world, LiveMath (CAS) with excellent symbolic manipulative functions, GSP and Cabri for easy construction and dynamic transformation of geometric figures, and EXCEL as a tool for various numerical problem solving. Because the computer can provide concrete methods to represent abstract and formal mathematics and particularly because its operation is under students' own control, if proper software is used, students can reflect on their own mathematical and cognitive activities and strengthen mathematical communication among students and teachers by representing their concrete operational activities through mathematical language.

Performance assessment

Recently, the evaluation system of mathematics has changed rapidly focusing on how to apply knowledge to a situation, on how to communicate a given situation and ideas concisely by using proper mathematical language, and on how to select and integrate information rather than on how much information students have. Under this direction, performance assessment has been introduced in order to evaluate what cannot be tested by the traditional paper-and-pencil test. Performance assessment is a test to evaluate an ability to set up an assumption, to check the assumption through practical or mental operations in various problem settings, and to examine the ability to communicate with other people in order to evaluate students' mathematical power rather than their memory of pieces of knowledge.

Perspectives on Algebra

Although many researchers define algebra differently (e.g., Kieren & Wagner, 1989; Usiskin, 1988), this paper defines algebra as a subject with the structure as presented in the figure 2. Algebra is a subject dealing with expressions with symbols and the extended numbers beyond the whole numbers in order to solve equations, to analyze functional relations, and to determine the structure of the representational system which consists of expressions and relations. However, activities such as solving equations, analyzing functional relations and determining the structure are not the purpose of algebra but tools for modeling of real world phenomena and problem solving related to the various situations.

Furthermore, algebra is much more than the set of facts and techniques. It is a way of thinking. Success in algebra depends on at least six kinds of mathematical thinking abilities as follows: Generalization, Abstraction, Analytic thinking, Dynamic thinking, Modeling, and Organization.

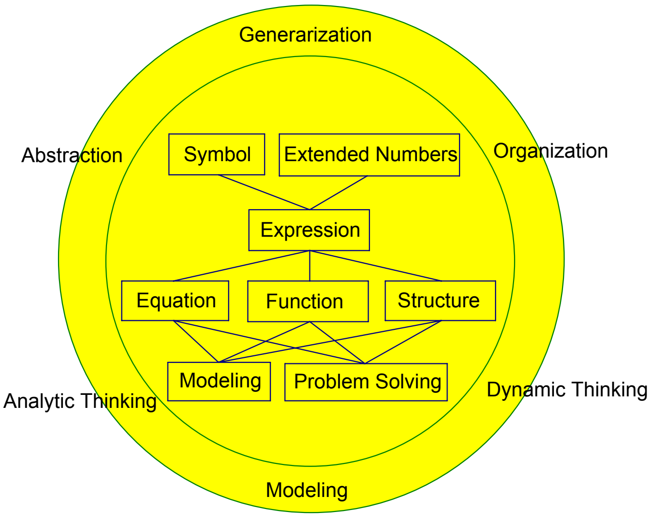

Figure 2. The Structure of Algebra

Generalization

Most of all algebraic objects or concepts are results of the generalization process. Generalization is a process to find a pattern or a form. Algebra begins with the patterns identified in the given set of objects. For example, the commutative property is just a pattern recognized in many numerical expressions like 1+2=2+1 and 10+20=20+10. Every functional relation is also a pattern. The linear function $y=2x$ is a pattern of the set {(1,2), (2, 4), (3, 6), ... } which shows for every ordered pair the former number is half of the latter number. But, the generalization generates physical knowledge by the so-called "*physical abstraction*" in a Piagetian sense. The physical abstraction contrasts well with the *reflective abstraction* by which a mathematical structure is extracted.

Abstraction

Unlike arithmetic, algebra deals with symbols. Symbol is an abstract object in the sense that it is de-contextualized. Abstraction is a process to extract mathematical objects and relations based on the generalization.

Abstraction in algebra is very different from abstraction in other subjects such as geometry. Seldom does algebraic language have concrete meaning. For example, the number 2 and variable x have no concrete meaning to accompany the image. In geometry, for example, a rectangle as an object abstracted from various rectangular shapes has some kind of image to describe the shape of rectangle. In algebra the symbol x does not evoke such an image. This characteristic is a serious stumbling block for students whose developmental ability has not reached the formal level for learning algebra.

When considering the abstraction process, although abstraction is opposite concreteness, it should be regarded as having a relative meaning. For example, numbers are abstract as well as concrete, depending on the compared objects.

Analytic thinking

The most important activity in algebra is the process of solving equations, which has a long history from the Babylonia era. Solving an equation is the process of finding some unknown value requested in the expression written in terms of the unknown value. In algebra, the unknown value x is regarded as an "*imaginarily*" known value. It will be solved "*actually*" by a series of necessary conditions. For example, consider the following equation:

$$\frac{2}{x^2-1}-\frac{1}{x-1}-1=0 \qquad (1)$$

The unknown value x is regarded as "known" in order to operate the unknown value. The necessary condition of (1) is obtained by multiplying the least common multiple of $x^2 - 1$ and $x - 1$ as follows:

$$x^2 + x - 2 = 0 \qquad (2)$$

Again the necessary condition of (2) is $x = 1$ or $x = -2$. This process is called *analytic* thinking. The reverse process, in the above example, is called *synthetic* thinking, to check that $x = 1$ is not the solution of the equation.

The working backward strategy is also a typical example of analytic thinking in the sense that it is a process to apply the inverse operations of the operations applied in the problem conditions in order to find a series of necessary conditions for the final outcome.

Dynamic thinking

The most important concept in algebra is the concept of variable. The variable is an object to encapsulate changing objects. Variable is essential for understanding a function. Dynamic thinking could be developed by hypothetical deduction and the trial and error strategy for monitoring and controlling the dependent action for each of changing variables. In the Greek period, the concept of variable did not exist in that they did not try to represent changing objects mathematically. As a result, there was no algebra in Greece.

Modeling

Modeling is a process to *represent* the complex situation using mathematical expressions, to *investigate* the situation with a model, and to *draw* some conclusions from the activities. This is the essence algebra and the reason to learn algebra. For example, when we teach an equation, it is important for students to represent some situation with an equation and to solve the equation to get the solution of the original situation.

Organization

One purpose for learning algebra is to develop a tool for organizing the complex situations by using a table and a diagram. Organization promotes combinatorial thinking for finding all of the independent variables, which is very important to many problem-solving activities. By sorting and organizing data by making a table, a whole picture about the problem situation and the relation between conditions of the problem can be grasped and the relation between an independent variable and the corresponding dependent variable can be controlled more easily.

Goals for Algebraic Thinking in the Elementary Level.

Korean textbooks provide various activities under its stated goals to improve the six kinds of thinking related to algebra. This section identifies the goals of developing algebraic thinking in the elementary level and shows mathematical problems related to each specific goal.

The specific goal for each of 6 kinds of thinking and the code for each specific goal are shown in Table 1.

G2 is higher than G1 in that as compared with G1 in the recognition level, G2 is in the application level of the patterns recognized. G3 is the highest because to use a simplification strategy, students need to deal with each of the simplified situations of the original one and to find a pattern of the situations.

Ab1 is higher than G1 because although Ab1 is based on G1, the former requires the pattern recognized as a result of the latter be checked if the pattern is applied to all possible mathematical range beyond a given problem situation. Ab2 and ab3 are related with the use of abstract symbols like □. Although numbers are also symbols, they are not further abstract in that students can handle numbers easily as concrete materials through many operational activities on numbers and operations.

Table 1.
The goal structure of algebraic thinking

Algebraic Thinking	Specific Goals	Code
Generalization (G)	Recognition pattern and relation from the series of numbers and figures	G1
	Solving problem using pattern discovered	G2
	Solving problem using simplification strategy	G3
Abstraction (Ab)	Understanding mathematical concepts and properties	Ab1
	Using symbols related with the concepts and properties	Ab2
	Operational activities with abstract symbols	Ab3
Analytic Thinking (An)	Solving equation by intuitive method	An1
	Solving equation by inverse operation	An2
	Problem solving using working backward	An3
Dynamic Thinking (D)	Separating one number in various ways and gathering them	D1
	Solving problem using the trial and error strategy	D2
	Identifying relation between two sets of changing objects	D3
	Solving problem using a direct proportionality	D4
Modeling (M)	Making a story related to a given expression	M1
	Making a problem related to a given expression	M2
	Posing a problem using a given expression with □	M3
	Modeling situation using a diagram or a figure	M4
Organization (O)	Sorting	O1
	Problem solving making a table	O2
	Problem solving using a logical deduction strategy	O3

Analytic thinking is related to the process used to get an unknown value. An1 and An3 relate respectively to thinking to solve an equation intuitively and by using the inverse relation of the inverse operations. An2 involves thinking to get an unknown value without establishing an expression. An3 is higher than An2 in that the latter is a natural thinking process, whereas the former is a formal thinking process.

Dynamic thinking is related to the dynamical manipulation of mathematical objects. D1 is the process of separating a given number in various ways, such as that which is necessary for regrouping of 10 in the addition and subtraction process. D2 is a strategy to solve a problem using the systematic checking process for each of all possible variables. D3 is the process to identify the relation between two sets of changing objects (variables) and to find the corresponding value of a variable for the specific known value of a given variable. This thinking might be a generalization process of higher level based on symbols by Ab2. However, this is assigned as dynamic thinking in that D4 is the process to use a direct proportion as a problem solving strategy, which is based on hypothetical deduction as a functional thinking.

M1 and M2 are, respectively, processes to make a story and a problem related to a given expression with the four operations +, -, x, ÷. They are a reversed process of the process to model situations using expressions with four operations +, -, x, ÷, which is not dealt with here because of being assumed as belonging to the area of numbers and operations. M3 is a process to model a situation using an expression with □. M4 is a process to model a situation using a diagram or a figure as a geometric model.

O1 is a process to organize a problem situation using a one-dimensional table. O2 is a process to organize a problem situation using a two-dimensional table. O2 is higher than O1.

Typical Problem Situations to Promote Specific Algebraic Thinking

This section shows typical sample problem situations that appear in the Korean elementary textbooks for achieving specific goals related to algebraic thinking and provided in the Table 1. The textbooks based on the 7^{th} national curriculum were developed by Ministry of Education and are being used in Korea exclusively without exception.

Generalization

Problem Solving with Simplification Strategy (G3): Seven 6th grade classes in Yunjung's school plan to have a volleyball league. How many games they will have? When the number of classes is 2, 3, or 4, investigate, respectively, the number of games. When the number of classes increases by one, how does the number of games increase? In the case of seven classes, how many games are there?

Abstraction

Operational activity with symbols (Ab3): Determine the values of A, B, C, and the □s in the following multiplication process.

```
      A B C
    × A B C
    ----------
     □ □ □ 9
     □ □ 4
   □ □ 1
  ---------------
  □ □ □ □ □ 9
```

The digit of the unit place value of C × C is 9. Guess the digit of C. In the case that the digit of C is 3, think about the following steps: The digit of the unit place value of AB3 × B is 4. What is the digit of B?; AB3 × B is a three digit number and the digit of the unit place value is 1. What is the digit of A?; In the case that the digit of C is 7, think about the following steps: The digit of the unit place value of AB7 × B is 4. What is the digit of B? AB7 × B is a three digit number and the digit of the unit place value is 1. What is the digit of A? 4. What are the digits of A, B, C?

Analytic Thinking

<u>Solving equations by the intuitive method (An1):</u> When □ is 8, what is the value of □ and □? What is □? What should be used to solve □, ○ □ ? What is ○, □, □ respectively? What is □? What should be used to solve □, □, □? What is □, □, □**?**

□+□=□,	○+○=□,
□+□+□=○,	□+□−□=□,
□+□−□=□,	□+□+□=□

<u>Working backward strategy (An3):</u> I bought a notebook for 480 won. And, I bought pencils using half of the remaining money. I bought an eraser using half of the remaining money. I have 180 won now. How much money did I have before buying the notebook? What is a question? Think about the money I had before buying the eraser. How much was the eraser? How much was the pencil? How much money did I have before buying the notebook?

Dynamic thinking

<u>Problem Solving with Direct Proportionality(D4):</u> The price of 2 bottles of cola is 1500 won. How many bottles could be bought for 8000 won? Complete the following table.

# of bottles	**2**	**4**	**6**	**8**	**10**	**12**
Price (won)	**1500**	**3000**				

What kind of relation is there between the number of bottles and the price of the cola? In the above table, between which two numbers is 8000 won located? Explain why. How many bottles could be bought for 10000 won?

Modeling

<u>Representing the situation using a diagram(M4):</u> How many of the o-mok games should be matched among 29 members if every member had a game with every other member? What is a question? If the number of students is 2, how many games are there? If the number of students is 3, how many games are there? If the number of students is 4, how many games are there? If the number of students is 5, how

many games are there? What rules are there? How many games are there among 29 members?

Organization

Sorting (O1): Seeing the pictures of children who are interested in reading as their hobby, answer the following questions. How many children are there who are boys with glasses? How many are boys without glasses? How many are girls with glasses? How many are girls without glasses?

Content Coverage for Algebraic Thinking in Each Grade Level

This chapter introduces the content coverage for algebraic thinking being taught in grades 1-6 divided into three categories of 1-2, 3-4, and 5-6; discusses some characteristics of each category; and finally provides the global trend of the whole grades through 1-6.

It should be understood that this analysis was not done on the areas of “number and operation”, “statistics and probability”, “geometry”, “measurement”. There are many algebraic aspects in geometry, examples of which are the transformation of figures and recognition of shapes. And, most of the activities in statistics and probability can be assigned into organization thinking. However, this study has focused on the content areas such as “pattern and functions” and “letters and expressions” which have been regarded as the area of algebra.

Table 2 shows the content coverage for algebraic thinking being taught in grades 1-2. Some characteristics are as follows: All of the six kinds of algebraic thinking activities are emphasized from the early grade level, which means that thinking is as important as operational skills in the 7th curriculum. The symbol □ to represent an unknown value is introduced from the first grade level. This activity might be difficult for the first graders when considering their cognitive ability. The process to solve for the unknown value □ intuitively is introduced. It is supposed to strengthen students’ calculation abilities.

Various modeling activities, including activities to relate a given expression to a situation, are provided. Activities to identify the mathematical structures like the commutative law of addition are introduced from the first grade. Various problem-solving strategies such as working-backward and making-a-table are introduced.

Table 3 shows the content coverage for algebraic thinking being taught in grades 3-4. Some characteristics are as follows: The activities beyond grades 1-2 are provided to continually strengthen various activities emphasized in grades1-2. Activities to deal with two or more objects simultaneously are introduced. For example, there are activities to determine some digits represented as □ or an alphabetic symbol in the vertical multiplication. Various problem-solving strategies such as simplification, and trial-and-error are introduced in the fourth grade.

The process to solve equations more formally by using inverse operations is introduced. The functional thinking to deal with the relation between two variables and to represent it in terms of symbols like □ and 0 is introduced. Modeling activities to use a diagram and a figure are introduced, which can be regarded as the "geometric" modeling process. This means that the range of representation tools becomes broader than numbers and symbols. Direct proportionality as a problem-solving tool is introduced. This is one of the signals that students' cognitive activities can be done on the hypothetic-deductive level.

Table 4 shows the content coverage for algebraic thinking being taught in grades 5-6. Some characteristics are as follows: Activities beyond grades 3-4 are provided to continually strengthen various activities emphasized in grades 1-2 and grades 3-4. Various problem-solving strategies such as simplification, working-backward and trail-and-error are dealt with frequently. A formal level of thinking like problem solving using a proportional expression is emphasized. Activities that deal with the synthesis of two ratios using a logical deduction strategy are also introduced.

Global trend of the grades 1-6

Table 5 shows the global trend of the grades 1-6 for teaching content to improve algebraic thinking. The some characteristics are as follows: All kinds of algebraic thinking are evenly emphasized throughout grades 1-6. This shows clearly the nature of the 7th curriculum with its main focus on the development of thinking. The level increases gradually. However, there seems to be no big difference in the level

Table 2.
Content coverage of grade 1-2

Thinking	Code	Activities	Grade - semester
Generalization	G1	Draw proper figures that identify a pattern they recognized.	1-1, 1-2, 2-1
		Paint color based on a pattern.	1-2
		Find various rules in the multiplication table.	2-2
	G2	Make the multiplication table of 12 ×12 based on the rules for the multiplication table of 9×9.	2-2
Abstraction	Ab1	Understand the commutative law of addition.	1-1
		Understand the relation between addition and subtraction.	1-2, 2-1
		Understand the commutative law of multiplication.	2-2
	Ab2	Use the symbol □ to represent an unknown value.	1-2
Analytic Thinking	An1	Solve an equation by finding necessary conditions.	2-1
	An2	Solve a problem using the working backward strategy.	2-2
Dynamic thinking	D1	Separate whole numbers less than 10 using various methods.	1-1
		Separate the number 10 using various methods.	1-2
		Separate the number 18 into three numbers.	2-1
Modeling	M1	Create a story that matches an expression involving addition and subtraction operations.	1-1, 1-2
	M2	Pose a problem using a given expressions involving addition and subtraction	2-1
	M3	Model a situation using an expression with □.	2-1, 2-2
Organization	O1	Sort a situation into cases using a certain criterion	1-1
	O2	Investigate a problem situation systematically by making a table to represent relationships among problem conditions.	2-2

Table 3.
Content coverage of grades 3-4

Thinking	Code	Activities	Grade-Semester
Generalization	G1	Make the figures of the fifth triangle and the fifth square number.	4-1
	G2	Determine the 13th figure due to the pattern discovered.	3-2
	G3	Find the number of equilateral triangles in the figure using a simplification strategy.	4-1
Abstraction	Ab1	Understand the inverse relation between multiplication and division operations.	3-1
	Ab3	Determine the value of some digits represented as □ or alphabetic symbols in vertical multiplication.	3-2, 4-2
Analytic Thinking	An2	Solve a problem using a working backward strategy.	4-2
	An3	Solve an equation by using the inverse relation of multiplication and division operations.	3-1
Dynamic thinking	D2	Solve problems using trial and error strategy	4-2
	D3	Recognize the age relation between two brothers.	4-2
		Represent the relation between two sets of numbers in terms of □ and 0.	4-2
	D4	Solve a problem using direct proportionality.	4-2
Modeling	M2	Pose a problem related to a given division expression.	3-1
		Pose a problem related to a given expression of mixed calculation	4-1
	M3	Model situations using an expression with □.	4-1
	M4	Model situations using a diagram.	3-1
Organization	O2	Organize a problem situation systematically by making a one-dimensional table to represent problem conditions.	3-1

Table 4.

Content coverage of grades 5-6

Thinking	Code	Activities	Grade - Semester
Generaliza-tion	G3	Solve a problem using a simplification strategy.	5-1, 5-2, 6-1, 6-2
Analytic thinking	An2	Solve a problem using the working backward strategy.	5-1, 6-1, 6-2
	An3	Solve for □ in a proportional expression like 3 : 4 = 9 : □.	6-1
Dynamic thinking	D2	Solve a problem using the trial and error strategy.	5-1, 5-2, 6-1, 6-2
		Make some figure using a tangram.	5-2
	D3	Represent the relation between a brother and a sister in terms of □ and 0.	6-2
	D4	Change two ratio between two persons to the continued ration among three persons.	6-2
Modeling	M3	Model a situation using proportional expression with □	6-1
		Model situations using multiplication expressions with □.	6-1
Organization	O2	Arrange all possible numbers with four digits using four numbers cards.	6-1
	O3	Solve problem using a logical deduction strategy on the two dimensional table.	5-1

of algebraic thinking between grades 3-4 and grades 5-6. This means that the teaching content is a burden imposed on the 3-4 graders. This problem should be improved in the next curriculum revision. Generalization is taught throughout the whole grades 1–6 step-by-step in stages. Dynamic thinking is particularly emphasized in grades 5-6. This might be helpful for teaching the concepts of variables and functions in the junior high school. However, abstraction is not dealt in grades 5-6 even though it is higher than generalization. In grades 5-6, instructional efforts to pay attention to the relation between numbers, operations and patterns reflecting basic skills and knowledge learned in grades 1-4 should be made.

Table 5.

Global trend of grades 1-6

4					M4			D4				D4
3			M3		An3	Ab3	G3 M3	D3 Ab3	G3 O3	G3	G3 An3 M3	G3 D3
2		Ab2	M2	G2 An2 O2	M2 O2	G2	M2	Ab2	An2 D2	An2 D2	An2 D2 O2	D2
1	G1 Ab1 D1 M1 O1	G1 Ab1 D1 M1	G1 Ab1 An1 D1	Ab1	Ab1		G1					
Level / Grade- sem.		1-2	2-1	2-2	3-1	3-2	4-1	4-2	5-1	5-2	6-1	6-2

New Directions for Promoting Algebraic Thinking

In the near future, the use of dynamic computer technology and the realistic approach should be discussed for promoting algebraic thinking in the elementary mathematics education.

Use of a spreadsheet for learning the concept of variables

The concept of variable is regarded as one of the most difficult and the most important in algebra. Much research done recently has tried to teach the variable concept through spreadsheet (e.g., Drier, 2001; Dettori, Garuti, & Lemut, 2001). They recommended use of a spreadsheet to represent and to investigate various mathematical ideas and situations related to the variables.

In using a spreadsheet, variable is learned by finding patterns through experience inputting numbers into cells of the spreadsheet. Each cell is assigned a name and each has some role of variable. The pattern discovery process gives students an experience to correct and reflect upon their performance and to use the results in the next set of calculations and procedures. Through active learning using a spreadsheet in applied problem solving and modeling, students can appreciate dynamic concepts of variables as changing, and abstract static and formalized concepts of variables as a generalized prevalent name.

Need for the more realistic situation

It is very positive that under the 7^{th} curriculum, various modeling activities are emphasized, which is in sharp contrast to the previous curricula. However, the problem is that materials used in the textbooks are very artificial and not natural, so that students do not think of mathematics as interesting. This is not matched well with a reason to learn the modeling process.

More realistic situations should be introduced in the textbooks in order to make students perform mathematical activities in the more interesting situations. Excellent models for more realistic approaches can be found in the reform textbooks published in the USA or the Netherlands.

Conclusion

Traditionally the Korean mathematics education society, under the skill-oriented curriculum, has focused on the low level skills such as solving equations, factoring polynomials in x, and drawing a graph of a function rather than higher level thinking such as modeling various situations using an expression to draw conclusions, applying mathematics in the real world situations, and abstracting mathematical structures. This kind of mathematics education has been criticized as making many students dislike mathematics and be afraid of applying mathematics. Particularly, teaching of algebra as a subject that deals only with abstract symbols confirms students' belief that mathematics is meaningless.

Since 1980, Korea has changed the tradition of mathematics education. Many mathematics teachers and curriculum developers have been trying to re-model mathematics classes in a rigorous way. The high achievement of Korean students in international competitions like TIMSS encourages the Korean educators to develop a better curriculum and teaching methods for students who will live in the highly developed information society. Most members of Korean society agree that various computational skills of Korean students that are superior to those of students in their countries' can be performed by technological devices like computers and calculators.

This article has summarized Korean perspective to improve algebraic thinking at the elementary level. Its main focus is on the development of thinking abilities. Algebra is a subject to learn a thinking way.

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